SPECTRAL DOMAIN TECHNIQUE FOR THE ANALYSIS OF WAVEGUIDE JUNCTION WITH ANISOTROPIC MEDIA

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Abstract

A rigorous analysis of a waveguide junction with lossy arbitrarily shaped anisotropic media is proposed. The analysis is based on the equivalence principle and the spectral domain technique and being dependent on neither the geometrical symmetry of the junction nor the number of ports. To demonstrate the validity of the method, H- and E-plane Y-junction circulators are considered. The influences of the magnetic and dielectric losses on the performance of the circulators are examined.

Introduction

The numerical characterization of waveguide junction is usually carried out by the point-matching method. Several authors have successfully used the method to analyse H- and E-plane Y-junction [1] - [6]. Okamoto [7] presented a method based on the integral equations derived from the reciprocity theorems for the analysis of H-plane waveguide junctions with full-height ferrites of arbitrary shape. Recently, the finite element method was used by Koshiba and Suzuki [8] for the analysis of H-plane waveguide junctions with full-height ferrite. In their approach, the magnetic loss is considered. This paper presents a method based on the equivalence principle and the spectral domain technique for the analysis of the waveguide junction with lossy arbitrarily shaped aniostropic media. Making use the method, the Y-junctions with ferrite and with coaxial composite ferrite are considered. The numerical results are compared with the earlier experimental results. The influences of the magnetic and dielectric losses on the performance of the circulators are examined.

Theory

Fig. 1 shows the waveguide junction with lossy arbitrarily shaped anisotropic media. Applying the equivalence principle, the imaginary boundaries chosen between the center region of the junction and the waveguides are short circuited and the magnetic surface currents $\underline{M}_s = \hat{\underline{n}} \times \underline{E}_g$ and $-\underline{M}_s$ are introduced at both sides of the inserted short circuits. $\underline{E}_g$ is the electric field of the waveguides. The center region of the junction can be treated as a resonator with magnetic surface currents $\underline{M}_s$ and anisotropic media. The fields inside the resonator satisfy Maxwell's equations

$$\nabla \times \underline{E} = -j\omega\mu_o \overleftrightarrow{\mu}(r)\, \underline{H} - \underline{M}_s \tag{1}$$

$$\nabla \times \underline{H} = j\omega\varepsilon_o \overleftrightarrow{\varepsilon}(r)\, \underline{E} \tag{2}$$

Since the electric displacement $\underline{D}$ is solenoidal in the resonator it can be expressed completely as an expansion in terms of a solenoidal set

$$\underline{D} = \varepsilon_o \sum_n e_n \underline{E}_n \tag{3}$$

where $\underline{E}_n$ is the electric field of the nth resonance mode in the empty cavity, e_n is the corresponding expansion coefficient. Due to the magnetic surface currents the magnetic induction $\underline{B}$ can be expressed in the resonator as an expansion in terms of a solenoidal set and an irrotational set

$$\underline{B} = \mu_o \sum_n h_n \underline{H}_n + \mu_o \sum_n g_n \underline{G}_n \tag{4}$$

where $\underline{H}_n$ is the magnetic field of the nth resonance mode in the empty cavity, $\underline{G}_n$ belongs to the irrotational set and h_n, g_n are the expansion coefficients.

Substituting (3), (4) into Maxwell's equations (1), (2) and making use the orthogonality properties of $\{\underline{E}_n\}$, $\{\underline{H}_n\}$ and $\{\underline{G}_n\}$

$$\varepsilon_o \int_V \underline{E}_p \cdot \underline{E}_q^* dV = \delta_{pq}$$

$$\mu_o \int_V \underline{H}_p \cdot \underline{H}_q^* dV = \delta_{pq} \tag{5}$$

$$\mu_o \int_V \underline{G}_p \cdot \underline{G}_q^* dV = \delta_{pq}$$

one obtains

$$\omega_n \varepsilon_o \int_V \underline{E}_n^* \cdot \overleftrightarrow{\varepsilon}^{-1}(r) \sum_m e_m \underline{E}_m dV = \omega h_n - j \int_S \underline{M}_s \cdot \underline{H}_n^* \, ds \tag{6}$$

$$\omega_n \mu_o \int_V \underline{H}_n^* \cdot \overleftrightarrow{\mu}^{-1}(r) \sum_m h_m \underline{H}_m dV$$

$$+ \omega_n \mu_o \int_V \underline{H}_n^* \cdot \overleftrightarrow{\mu}^{-1}(r) \sum_m g_m \underline{G}_m dV = \omega e_n \tag{7}$$

$$g_n = \frac{j}{\omega} \int_S \underline{M}_s \cdot \underline{G}_n^* \, ds \tag{8}$$

where asterix denotes complex conjugate, ω_n is the nth resonant frequency of the empty cavity, ω is the frequency of the driving field.

Continuing the tangential magentic fields over the imaginary boundaries of the junction

$$\hat{\underline{n}} \times \underline{H}_g^* \Big|_{\text{imag. boundaries}} = \hat{\underline{n}} \times \underline{H}^* \Big|_{\text{imag. boundaries}} \tag{9}$$

and testing equation (9) with the eigenvectors of the waveguide modes one obtains the following linear matrix equations of infinite dimension

$$\left[C^{ee}\right] \underline{e} = \omega \underline{h} - \left[R\right] \underline{v} \tag{10}$$

$$\left[C^{hh}\right] \underline{h} = \omega \underline{e} - \left[C^{hg}\right] \underline{g} \tag{11}$$

$$\underline{g} = \left[S\right] \underline{v} \tag{12}$$

$$\left[P^*\right] \underline{v} = \left[RR\right]^\dagger \underline{h} + \left[SS\right]^\dagger \underline{g} \tag{13}$$

where † denotes complex conjugate and transpose.

The column matrixs $\underline{e}$, $\underline{h}$, $\underline{g}$ and $\underline{v}$ can then be solved. The elements of the column matrix $\underline{v}$ are the amplitudes of the waveguide modes. For numerical results the matrix equations are truncated and the resultant finite equations solved by digital computation.

Numerical Results

To demonstrate the validity of the theory we consider the H- and E-plane Y-junctions with a central circular ferrite and a composite ferrite post. The composite ferrite, that is the combination of a ferrite and a dielectric sleeve, offers advantage in high power application since the thermal conductivity of the dielectric material is usually higher than that of the ferrite. The numerical results are carried out by using 7 and 40 expansion functions in circumferential and radial direction, respectively, and 3 waveguide modes for the H-plane 11 waveguide modes for the E-plane Y-junction. Figs. 2-4 show the calculated results. The results for the Y-junction with lossless and lossy ferrite are represented by the solid lines and the triangles, respectively. The experimental results of [3] and [5] are represented by the dashed lines. As can be seen the calculated results agree well with the experimental results. The differences of the calculated results for the lossless and lossy ferrite are small with the exception of f = 10 GHz in Fig. 2.

Conclusion

A method based on the equivalence principle and the spectral domain technique for the analysis of the waveguide junction with lossy arbitrarily shaped anisotropic media was proposed. The validity of the method was confirmed by comparing the numerical results for the H- and E-plane circular rite and composite ferrite post circulators with previously published experimental results. The influences of the magnetic and dielectric losses of the used ferrite materials on the performance were very fluences of the magnetic and dielectric losses of the used ferrite materials on the performance were very small. The present method can be readily used for the analysis of circulators with partial-height ferrite insert and radial transformers. The numerical data for such cases will be reported in near future.

References

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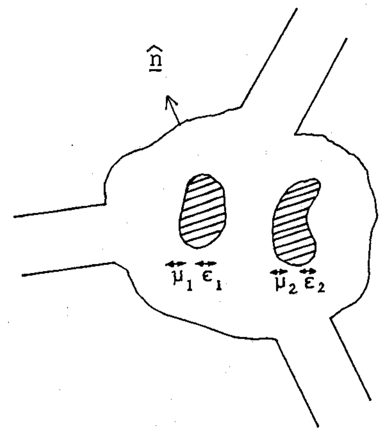

Fig. 1. A waveguide junction with anisotropic media.

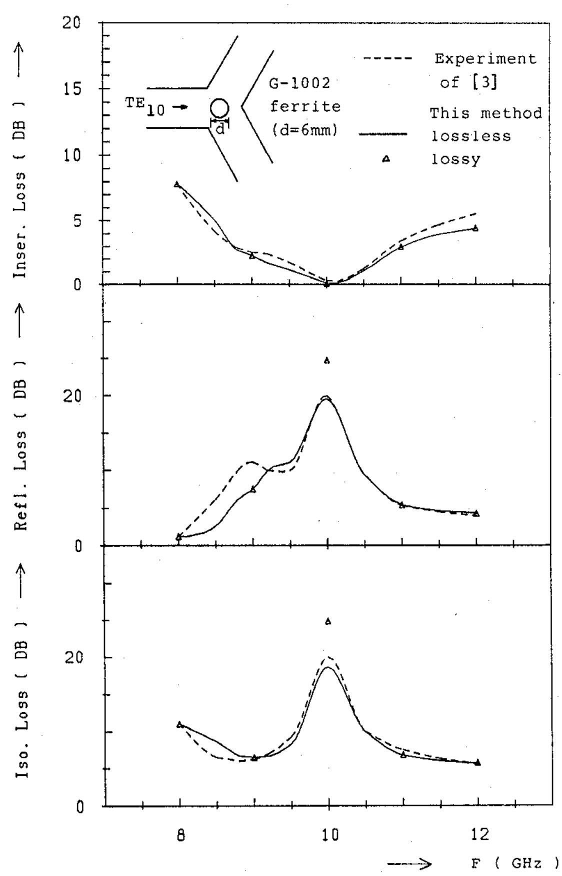

Fig. 2. Performance of a H-plane Y-junction with a G-1002 circular ferrite post.

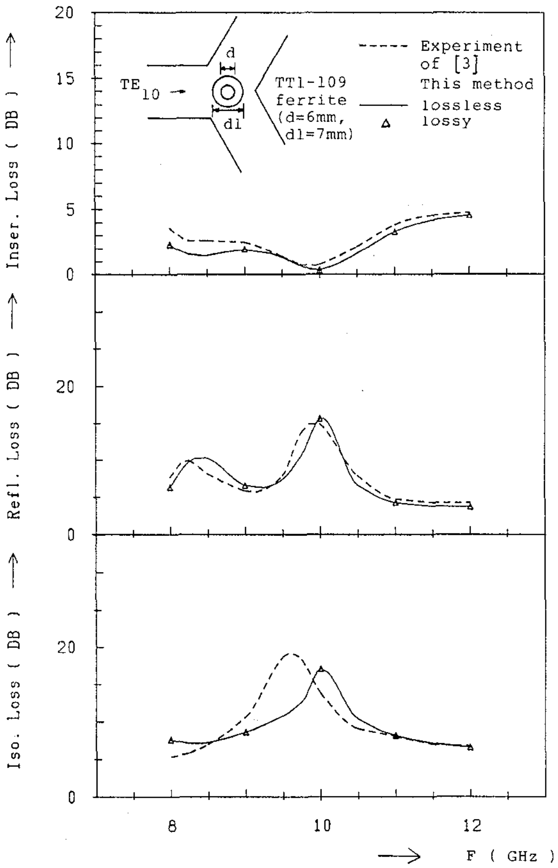

Fig. 3. Performance of a H-plane Y-junction with a TT1-109 ferrite rod and a dielectric sleeve(ε_r =12.9, tan δ =0).

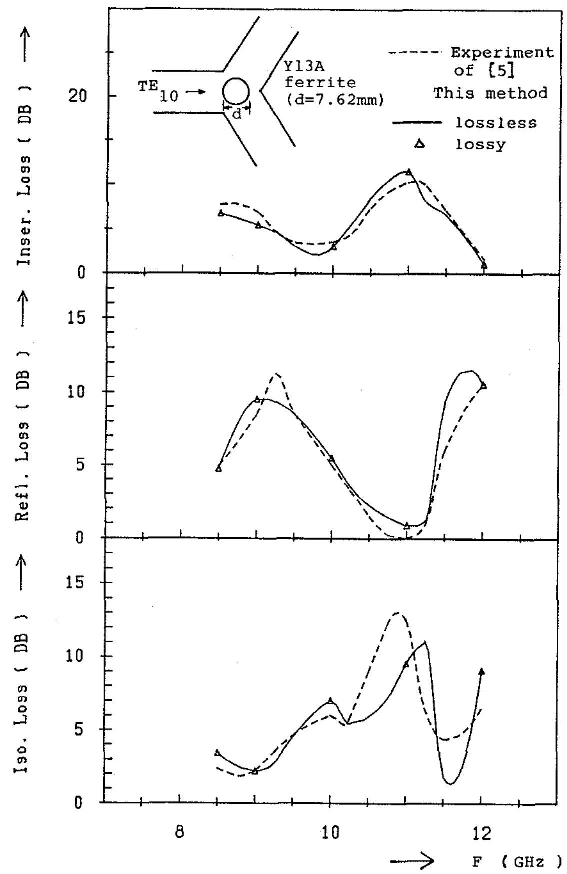

Fig. 4. Performance of a E-plane Y-junction with a Y13A ferrite rod.